Do strain rates determine the spatial density of crevasses on the Greenland Ice Sheet?

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***Abstract*— To compare spatial crevasse density with an existing strain rate dataset, a Fast Fourier Transform (FFT) algorithm was used to create a one dimensional spatial crevasse density map from a 2.25 km^2 area on the western flank of the Greenland Ice Sheet (GIS). Although we found a poor correlation between crevasse density and longitudinal strain rates, the correlation improved significantly when the crevasses were projected five years upstream. This suggested that the crevasse patterns were relicts of strain rates the ice felt five years ago, and that it took five years for crevasses in the study area to open fully. The stress required to create these crevasses, 111 ± 47 kPa, compares well to the existing body of literature on tensile strength. The average total crevasse life span of twelve years in the study area region was found to vary greatly from that on the Worthington Glacier in Alaska, where crevasses persist for only one to two years.**

Keywords- spatial crevasse density; strain rate; tensile strength;Fast Fourier Transform algorithm; Greenland Ice Sheet; crevasse life span; structural glaciology

I. Introduction

Glaciers are massive bodies of ice that slowly flow due the effects of gravity. As the ice flows, it diverges and converges in various areas due to the effects of the topography steering it. Strain rate, the measure of gradients in deformation, is used to measure this spatial divergence of ice as it flows. The strain rate of the ice in a certain region is a product of the amount of stress in that area; if the stresses are high enough, the resulting strain can fracture the ice at the surface of the glacier, creating formations called crevasses. Research has found that the pattern and orientation of crevasses on fast-moving ice masses can be predicted by the local strain rates [1].

How do strain rates affect the spatial density of crevasses? This project compares these two quantities to find a relationship in a heavily crevassed area of the Greenland Ice Sheet (GIS), a region which has not been previously explored in the crevasse literature. We also calculate the crevasse life span relative to the area of study and the tensile strength of these crevasses to contribute this information to the body of calculations that have been done for other glaciated areas.

II. Methodology

A. Selecting the area

To compare the spatial crevasse density with an existing strain_rate dataset, we analyzed photographs of the ice surface on the western flank of the GIS taken by a commercial satellite, operated through DigitalGlobe.com. These geo-located images have a pixel resolution of approximately 39 centimeters captured in the visible spectrum. From these images, we analyzed and created a map of the spatial density of crevasses in a 2.25 km^2 area of the ice sheet. In order to extract the maximum amount of information about the crevasses in these images, we found an area with the least interference that would affect with the calculations of crevasse density. We chose this study area based on the quality of the photograph there: it featured few clouds, clearly defined crevasses, and few melt water lakes. We removed one large body of melt water present in this image because it covered crevasses and would interfere with the crevasse density calculations. The chosen photograph was taken on August 11, 2008. The center of the study area is at approximately N 68° 44’ 27”, W 49° 58’ 33”, about 50 km southwest of the calving front of Jakobshavn Isbrae, as shown in Fig. 2.

We developed a Fourier transform algorithm to calculate spatial crevasse density over a one-dimensional transect. In general, a Fourier transform is a mathematical operation that can decompose any waveform into a series of fundamental sinusoids, or waves [2]. It is often applied in signal processing to identify the frequency of an original signal that is hidden in noise. Images themselves can be interpreted as a combination of waves. Every digital image is composed of a series of pixels that each contains its own intensity value (lightness or darkness). These discrete values of image intensity can be thought of as a spatial waveform, and a Fourier transform algorithm can evaluate the waveform’s fundamental wavelengths or frequencies. For a region of crevasses, this algorithm is useful for understanding the representative wavelengths between crevasses, and thus indirectly the density of crevasses in a certain area.

B. *Crevasse Density*

In order to check the general accuracy of the Fourier transform algorithm in returning the spatial density of crevasses, we compared it to directly counting the number of crevasses across a transect. Both types of calculation are dependent on the derivation of one-dimensional spatial crevasse density, a metric which we developed. We define one dimensional crevasse density (D_1) to be the ratio of the quantity of crevasses intersecting a one-dimensional transect (Q_c) and the length of that transect (L).

$$D_1 = \frac{Q_c}{L} \tag{1}$$

We draw this transect perpendicular to the dominant local crevasse orientation, which is also generally parallel to the local velocity vector of the ice. To automate the process of calculating D_1, we analyze the pixel intensities of crevasses along this line in a Fast Fourier Transform (FFT) algorithm in Matlab to return the principal component of the spacing between the crevasses (λ_{FFT}). This wavelength represents the typical length span across the fissure of the crevasses and thus can be used to determine the quantity of crevasses along a line. This is done by dividing the total length of the line by the crevasse spacing wavelength, thereby revealing how many crevasses should exist along that line.

$$Q_c = \frac{L}{\lambda_{FFT}} \tag{2}$$

The one-dimensional crevasse density simply equals the reciprocal of the FFT wavelength, which is measured in inverse meters. Fig. 1 illustrates this concept in an aerial and cross sectional view.

$$D_1 = \frac{\frac{L}{\lambda_{FFT}}}{L} = \frac{1}{\lambda_{FFT}} = \lambda_{FFT}{}^{-1} \tag{3}$$

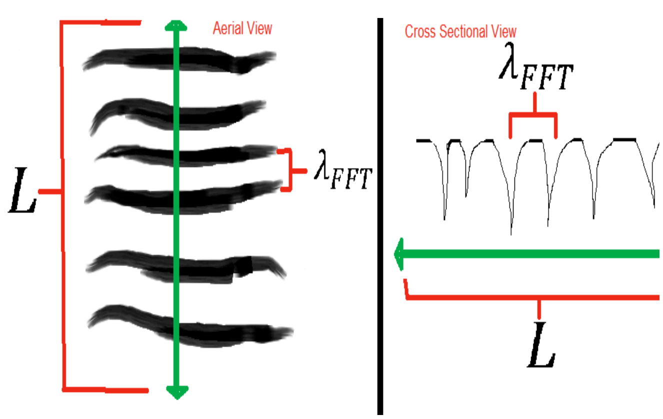

Figure 1. An aerial (left) and cross sectional (right) illustration of crevasses. The wavelength returned by our fast Fourier Transform algorithm, λFFT, represents the usual spacing between these adjacent crevasses while L represents the length of the transect (in green) intersecting these crevasses.

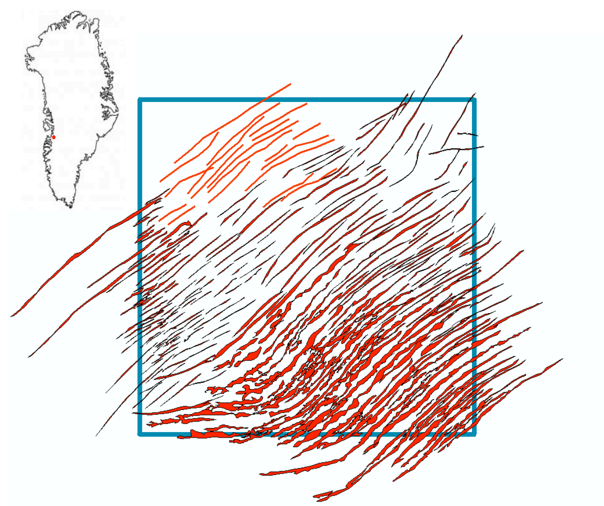

Figure 2. Using a satellite photo (not shown) located at N 68° 44' 27", W 49° 58' 33" (see inset), we digitized well-defined crevasses (dark red) within the 2.25 km^2 study area (blue outline). The red lines located in the northwest of the region indicate the general orientation of poorly defined crevasses. The spatial density of these hand-digitized crevasses are be used to check the accuracy of the Fast Fourier Transform algorithm for efficiently determining crevasse density over larger areas.

We compare the output of the Fourier transform algorithm to a direct count of the number of clear crevasses intersecting each transect, per (1). To better count these crevasses, we digitized, or outlined, their openings in ArcGIS. These digitized crevasses existed within a 1.5 by 1.5 kilometer area. Poorly defined crevasses exist in the northwest corner of this area; they were difficult to digitize accurately so we traced lines over these crevasses to display their general orientation. Fig. 2 illustrates the digitized crevasses mapped over our 2.25 km^2 study area.

C. *Creating the Crevasse Density Map*

We created a crevasse density map in the same area as the digitized crevasses. The process behind creating this map required various steps that divided the regional area into sub-areas or cells, calculate each cell's crevasse density, and recombine those calculations into a map. We divided the area into a 7x7 matrix of 49 cells with equal sides of approximately 214 meters, with each cell as a pixel of the crevasse density map. Within each cell, we used ArcMap to draw a line at least 180 meters long and generally perpendicular to the crevasses. To help properly position these lines within the cells, we overlaid a checkerboard-patterned grid with pixel sizes equal to the size of the crevasse density cells, as shown in Fig. 3. We analyzed the darkness of each pixel along these lines in Matlab with the FFT algorithm. This process returned a matrix containing the spatial crevasse densities for each cell.

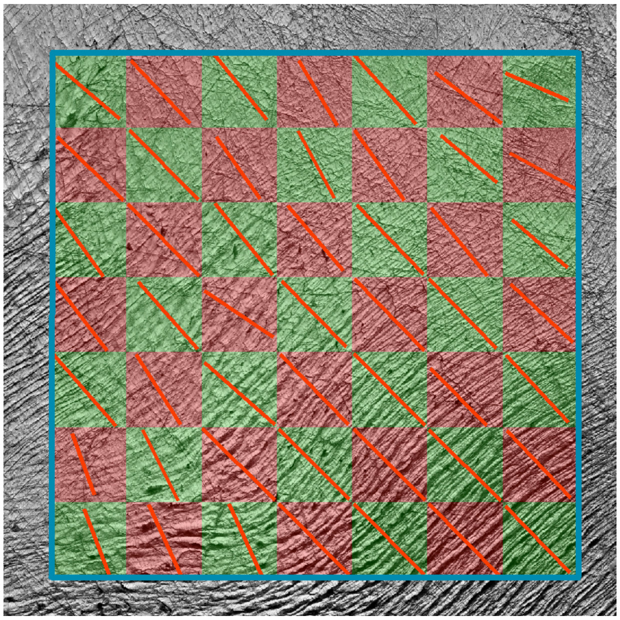

Figure 3. A checkerboard-patterned grid overlayed on the study area (blue) to properly draw transects (red lines) across the crevasses in each cell. The pixel intensities (lightness and darkness) along these lines were used in the FFT algorithm to calculate spatial crevasse density.

D. *Calculating Crevasse Life Span*

Along some flowlines in this area, we found crevasses transitioning from well-defined with clear and crisp edges (area 1), to poorly or vaguely defined with blurry edges (area 2), and then disappearing altogether (area 3). We identified the approximate central nodes of each of these three areas and calculated the travel time of the ice through these areas by finding the distance between these nodes and dividing by the average velocity at each node, as shown in (4). We calculated the local life span of crevasses in several regions, as shown in Table I in the appendix.

$$\frac{\text{Total Distance between area 1 and area 3}}{\text{Ice Velocity}} = \text{Life Span} \tag{4}$$

E. *Tensile Strength*

Tensile strength is the maximum amount of stress ice can endure while still behaving plastically. When stresses exceed the tensile strength, the ice fractures, forming crevasses. The tensile strength is a function of longitudinal and transverse deviatoric stresses, per (5) and (6) for longitudinal and transverse deviatoric stresses respectively [2]. We used a value of $1.4\text{x}10^{-17}$ Pa^{-3} yr^{-1} for the softness parameter A, which is a function of the temperature of ice at the depth of crevasse formation. We estimate this temperature to be -10° C.

$$\sigma_L'^3 = \frac{2\dot{\epsilon}_L^3}{A(\dot{\epsilon}_L + \dot{\epsilon}_T)^2} \tag{5}$$

$$\sigma_T'^3 = \frac{2\dot{\epsilon}_T^3}{A(\dot{\epsilon}_L + \dot{\epsilon}_T)^2} \tag{6}$$

We calculated the tensile strength of the ice in crevassed regions using two different methods, the von Mises fracture criterion (7) and the maximum strain-energy dissipation criterion (8), which are two different theories commonly used for determining tensile strength [2].

$$\sigma_{ts_{me}} = \sqrt[4]{(\sigma_L'^2 + \sigma_T'^2)(\sigma_L'^2 + \sigma_T'^2 - \sigma_L'\sigma_T')} \tag{7}$$

$$\sigma_{ts_{vM}} = \sqrt{\sigma_L'^2 + \sigma_T'^2 - \sigma_L'\sigma_T'} \tag{8}$$

III. Results

A. *Tensile Strength*

Fig. 4 shows a histogram of tensile strengths calculated by both the von Mises and maximum strain energy methods. Though the methods have different distributions, they encompass a similar mean and outline a broadly smooth envelope. The mean tensile strength for crevasse formation in the study area is 111 ± 47 kPa. The range of tensile strengths collected from the crevassed area in this region closely match the range compiled by Vaughan, which span 90 to 320 kPa [2]; the range in our study area was from 50 to 300 kPa. Overall, the values calculated in the western flank of GIS can be added to this compilation.

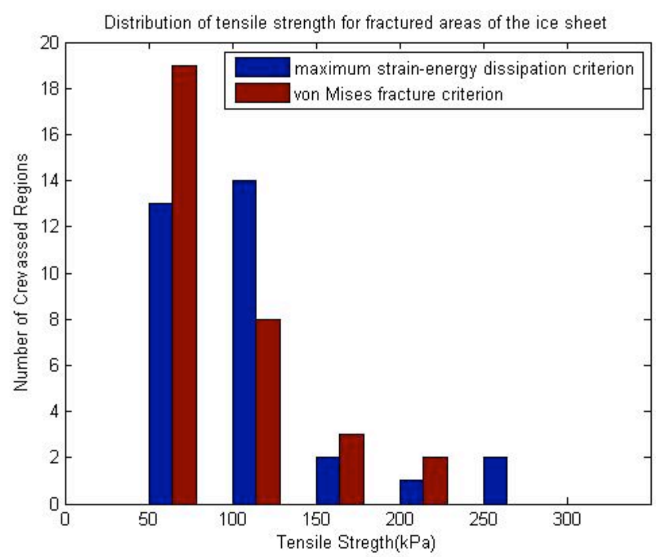

Figure 4. A histogram displaying the tensile strength of several fractured areas on the western flank of the GIS, calculated using the maximum strain-energy dissapation and von Mises fracture criterion methods.

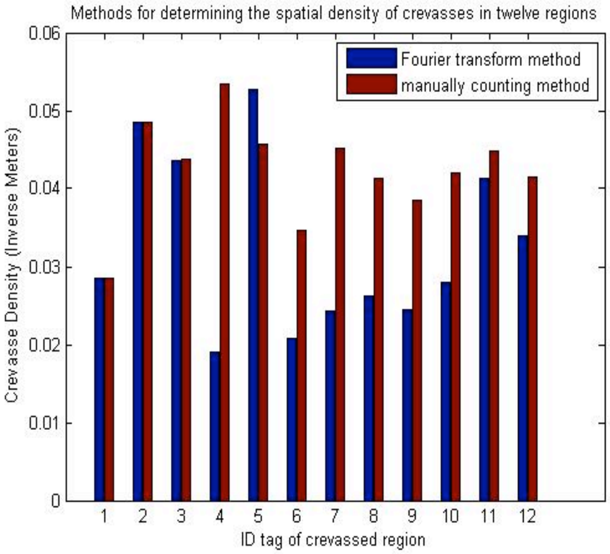

Figure 5. A bar graph comparing the Fourier transform and Manual method of spatial crevasse density calculation within 12 cells located near the southeast corner of the study area, where crevasses were the most well-defined.

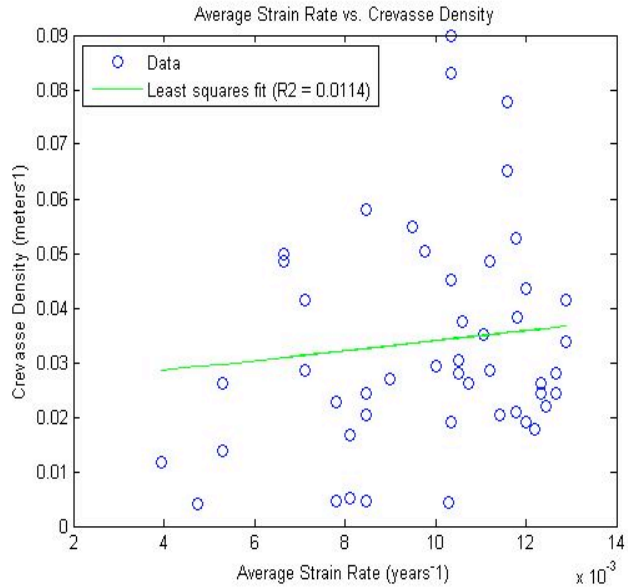

Figure 6. The spbatial crevasse density map shows a maximum R^2 or goodness of fit with strain rates experienced five years ago.

B. *Crevasse Density Map vs. Strain Rate Map*

We analyzed the fitness of our Fourier transform algorithm for automatically determining the spatial crevasse density. Fig. 5 illustrates this comparison from twelve cells located near the southeast corner of the study area. These cells were the best areas for digitizing crevasses because they had well-defined crevasses. We found that the FFT algorithm's results systematically underestimate the crevasse density by 22% when compared to manually counting. The way in which the crevasses were digitized may explain this error. Unlike the manually digitized crevasse outlines, the FFT algorithm returns the wavelength that represents the most variance in the data. This wavelength in the crevasse density equation sets a uniform crevasse spacing across the path that is usually larger than most of the spacing between manually digitized crevasses, perhaps because we were able to recognize thin, light gray cracks as crevasses by eye, but they were not dark enough for the FFT to differentiate them as being significantly different from the white ice sheet background. Since the crevasse spacing is inversely proportional to crevasse density, the FFT algorithm's values are lower than that of the manual method.

We compared the crevasse density results to the average of the strain rate values within each cell of the crevasse density map. We also compared the crevasse density results to the strain rates that affected the crevasses approximately two, five, and seven years ago. We did this by projecting the crevasse density map upstream by the distance traveled in these time spans. Out of all comparisons, the correlation between crevasse density and the strain rate experienced five years ago had the maximum correlation coefficient R^2, or goodness of fit, as shown in Fig. 6.

IV. Discussion

Even though all four comparisons of strain rate to crevasse density had a poor fit to a line of linear trend, the R^2 of strain rate five years ago was the highest and 3.5 times larger than the second highest R^2 value, which originated from the current crevasse locations. This suggests that crevasses in this region may have been influenced by strain rates approximately five years in the past; this is when expanding stresses in that area exceeded the ice sheet's tensile strength, which resulted in the slow formation of these crevasses.

Table I shows the life spans of crevasses taken from five different regions, which includes the location of the nodes, their velocities and the life span for each set. The average life span is approximately 12 years. This differs greatly from the life spans observed on the Worthington Glacier in Alaska, which only last 1-2 years [1]. The difference in topography of these two areas may exert a strong influence, since perturbing movements occur more slowly in polar regions with smoother surface and bedrock topography than they do in alpine regions with steeper, more rugged slopes. This could also account for the five year offset between crevasse density and strain rate data. This explanation is supported by the illustration in Fig. 7 which compares the outlines of currently crevassed areas to those shifted five years upstream along the ice's trajectory. These dotted outlines seem to match with areas of along-flow extension (positive longitudinal strain rate). A few crevassed areas exist in areas of longitudinal compression; these are outlined in red in Fig. 7. The crevasses in these regions are positioned nearly perpendicular to the trajectory; they are areas of high transverse strain rates. This suggests that in these few isolated areas, crevasses have opened in response to highly extensive transverse stress.

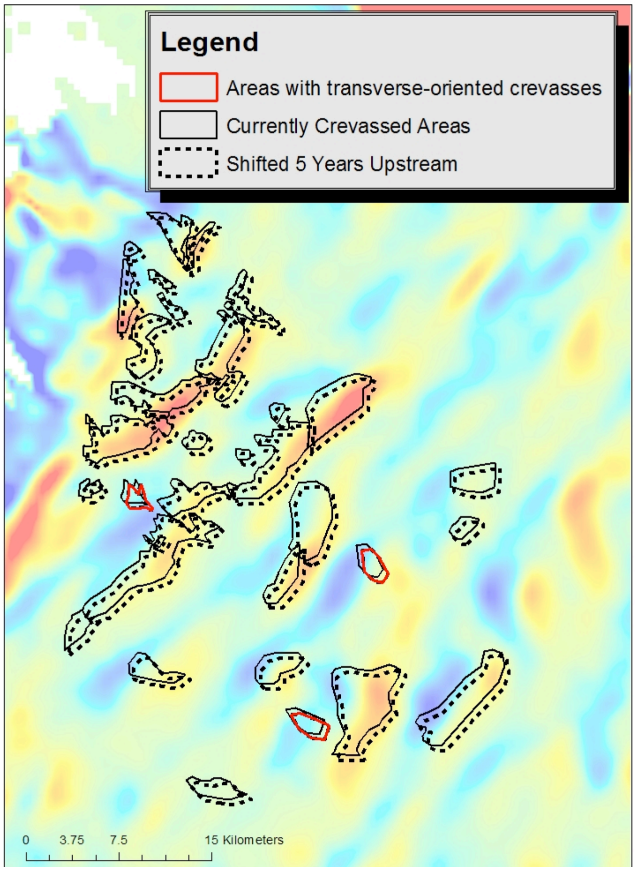

Figure 7. Map of the locations of highly-crevassed regions superimposed on the longitudinal strain rates in the study area. Areas outlined in solid black show where crevasses currently exist while the areas outlined in dashed lines show where they were five years ago. These areas shifted five years upstream match patterns of along-flow extension (longitudinal strain rate) which are plotted in yellow and red. The areas outlined in red represent transverse oriented crevasses and correlate to areas of along-flow compression (transverse strain rate) which appear as blue and indigo.

V. Conclusion

The average tensile strength in the study area on the western flank of the GIS has been found to be 111 ± 47 kPa and can now be added to the compilation of tensile strengths of various areas begun by Vaughan [2]. The difference in crevasse lifespan of the GIS and the Worthington Glacier in Alaska implies that crevasses behave differently in various geographical regions and that ice responds to the imposed strain rate faster in alpine regions than on the GIS. Although we find a poor correlation between spatial crevasse density and the local strain rate, we calculate a maximum in goodness of fit of crevasse density with strain rate data projected five years upstream. This suggests that crevasses on the GIS take approximately five years to open and respond fully to extensional strain rates.

VI. Future Works

There is a need for higher resolution strain rate datasets. We could only make broad comparisons between crevasse density and strain rates because the strain rate's resolution was relatively low; this may have caused such the poor correlation. Higher resolutions would reveal higher detail that could elucidate new understanding of crevasse patterns and formation mechanisms. Highly resolved images of the bedrock may also spawn new research exploring its relationship to crevasse formation. There is also a need to explore more crevasses in different areas of the GIS to check for variations in the results presented here.

VII. Acknowledgements

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VIII. References

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IX. Appendix

TABLE I. CREVASSE LIFESPANS IN THE WESTERN FLANK OF THE GREENLAND ICE SHEET

Crevasse Families	Location	Descriptions/Tags	Velocity (Meters/Year)	Life Span (years)
Family 1	49°53'9.73"W 68°41'10.476"N	Well-defined	152	11
	49°53'46.323"W 68°41'35.368"N	Vaguely defined	144	
	49°54'22.114"W 68°41'55.76"N	Non-existent	148	
Family 2	49°51'26.437"W 68°43'54.905"N	Well-defined	136	6
	49°52'11.839"W 68°44'17.061"N	Vaguely defined	134	
	49°52'39.287"W 68°44'34.777"N	Non-existent	130	
Family 3	50°2'21.913"W 68°37'49.074"N	Well-defined	155	15
	50°1'16.622"W 68°37'19.933"N	Vaguely defined	157	
	50°0'0.877"W 68°36'50.865"N	Non-existent	157	
Family 4	49°44'10.772"W 68°41'59.67"N	Well-defined	132	10
	49°44'48.519"W 68°42'16.963"N	Vaguely defined	123	
	49°45'17.305"W 68°42'31.966"N	Non-existent	115	
Family 5	49°40'11.809"W 68°37'45.311"N	Well-defined	142	17
	49°41'39.407"W 68°38'2.485"N	Vaguely defined	121	
	49°42'59.407"W 68°38'23.118"N	Non-existent	129	